

# **Torsional rigidity in beams with arbitrary cross-sections**

*A Thesis Submitted in Partial  
Fulfilment of the Requirements for  
the Degree of*

**Bachelor of Technology  
in  
Mechanical Engineering  
by**

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**May 2014**



**NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA**

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## CERTIFICATE

This is to certify that the research work that has been presented in this thesis entitled “Torsional rigidity in beams with arbitrary cross-sections” by **Sudeep Nayak**, has been carried out under my supervision in partial fulfilment of the requirements for the degree of *Bachelor of Technology in Mechanical Engineering* during session 2013-2014 in the *Department of Mechanical Engineering, National Institute of Technology Rourkela*.

As per my knowledge this dissertation work has not been submitted in any other college or university at any time prior to this, for the award of any degree or diploma.

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Date: 12/05/2014

## **ACKNOWLEDGEMENT**

I would like to take this golden opportunity as a privilege to thank all individuals without whose support and guidance I could not have completed our project in this stipulated period of time. I, wholeheartedly, acknowledge the intellectual stimulation of my esteemed guide Dr. Manoj Kumar Moharana, (Assistant Professor, Mechanical Engineering Department) for his continuous help and guidance throughout the project in spite of his busy work schedule. He helped me out in understanding the subject well & he always stood beside me whenever I needed his help. It is the result of his sincere of co-operation that I finished my project in stipulated time

I am highly indebted to Department of Mechanical Engineering for providing necessary information and guidance regarding the project and also for their support in completing the project.

And last but not the least, I would like to thank my friends and my family for their support.

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## **Abstract:**

In solid mechanics, when a torque is applied on a body, the body twists. This is called torsion. In those sections which are perpendicular to the torque axis, the resultant shear stress is in a direction, perpendicular to that of the radius. Almost all the rotating parts of a machine experience torsion and an accurate study of torsional study is essential for proper designing of machine components so as to have desired physical properties . Structures with different kind of cross-sections have different values of torsional rigidity. But currently, work has been done to accurately find out the torsional rigidity of only standard beam structures (like hexagonal etc.). In the present work, a semi analytical boundary collocation method has been used to predict the torsional rigidity of any arbitrary shaped cross-sectional beams.

## **Introduction:**

In the designing of structural elements, studying torsion is very important and quite basic. We may formulate the problem in Laplace equation involving boundary values in case of a wrapping function, or Poisson equation involving boundary value in the Prandtl's stress function. There exists exact solutions for standard cross sectional shapes like circular, triangular, elliptical, annular and regular polygonal. With developments in computational capabilities, numerical techniques can be used as an alternate for both standard cross-sectional as well as complicated shapes. The boundary element method, the finite element method or finite differences method are the most widely used methods. In the last two of the methods, complete cross-section needs to be discretized to elements or grids. In order to make it accurate, it would require a large number of nodes and elements for a complicated shape. Here, boundary collocation method comes into picture. In this method, only the boundary needs to be discretized instead of the whole cross-section. A fast development in the mesh free methods have been seen in the recent years. There are many of these mesh free methods.

One of these mesh free methods is the Trefftz method. In this method, the solution of the partial differential equation is expanded in respect of T-functions or the trial functions of the equation, as a result, that equation is identically satisfied by the proposed approximate formula. We now apply the boundary conditions in an approximate manner in order to determine the included expansion parameters. Boundary collocation method is one of the simpler versions of the aforementioned procedure. In this method boundary conditions are satisfied only at finite points along the boundary. Hence it is considered as a semi analytical method as it provides analytical expressions which are accurate in nature, which provide solutions through simple numerical calculations.

Boundary collocation method (BCM) has been applied by many authors for solving torsion problems. There has been extensive work on such problems for solid bars having regular cross sections like circular or polygonal. But relatively less work has been done on torsion problems in complicated cross sections and even if some studies have been done, they take up the problems individually. So there needs to be a generalization of such solutions.

This work involves the torsion in rods whose cross-section is arbitrary in nature. The objective of this work is to present a generalized solution that can be applied to bars having any kind of cross-section, with or without holes of any arbitrary shape, provided that, it has at least one plane of symmetry. The theoretical analysis results in a situation where one requires to solve a set of linear equations with a set of unknowns, thus requires use of numerical methods such as Gauss Seidel technique. In the present work, MATLAB is used as a tool for this purpose.

### **Motivation behind the current work:**

It was seen that the governing differential equation for stress functions are very similar to that of the heat conduction as well as the fluid flow equations. Hence the stress equations can be efficiently worked on parallel lines with the heat flow or the fluid flow equations, and that is what has been done. The work includes:

- Studying the heat diffusion equations and the application of boundary collocation technique for solving them
- Analyzing the BCM technique in lines with fluid flow problems
- Studying the generalization of this technique for any arbitrary cross section
- Generating governing differential equations for various cases viz. solid arbitrary cross-section, arbitrary cross-section with cylindrical hole and cylindrical bars with arbitrary hole.
- Using different boundary conditions to simplify and solve them using MATLAB.
- Using MATLAB to generate coordinates of the profile and calculate stress function values.
- Generating a contour plot from those values.

## **Literature survey:**

Boundary collocation method has been used many a times in previous works, quite efficiently. This method has been used in finding non dimensional torsional rigidity, in heat and fluid equations as well. Kołodziej and Strek [1] worked on the shape factor concept in regular polygons. For solving relevant boundary value related problems, they used the boundary collocation technique involving least square approximation. They also presented elementary analytical formulas which gives shape factors for the geometries that they have considered. Kołodziej and Fraska [2] found out the non-dimensional torsional rigidity in regular polygonal beams by integrating the equations of stress functions analytically for the several cases that have been taken into account, the non-dimensional stiffness of rods were calculated using an analytical formula. Numerical results were also shown for various cross-sectional shapes so as to show the accuracy as well as the efficiency of the BCM. Moharana and Das [3] analyzed conduction through shaped tubes with circular inner surface and hydrodynamically shaped outer surface by two different techniques. They made a two-dimensional analysis by a semi analytical technique using boundary collocation at the outer periphery. Moharana and Das [4] used boundary collocation technique to find the heat flow through an eccentric insulation surrounding a circular cylinder. Moharana and Khandekar [5] presented a generalized form for finding pressure drop in any arbitrary shaped channel which may be singly or doubly connected using the boundary collocation method.

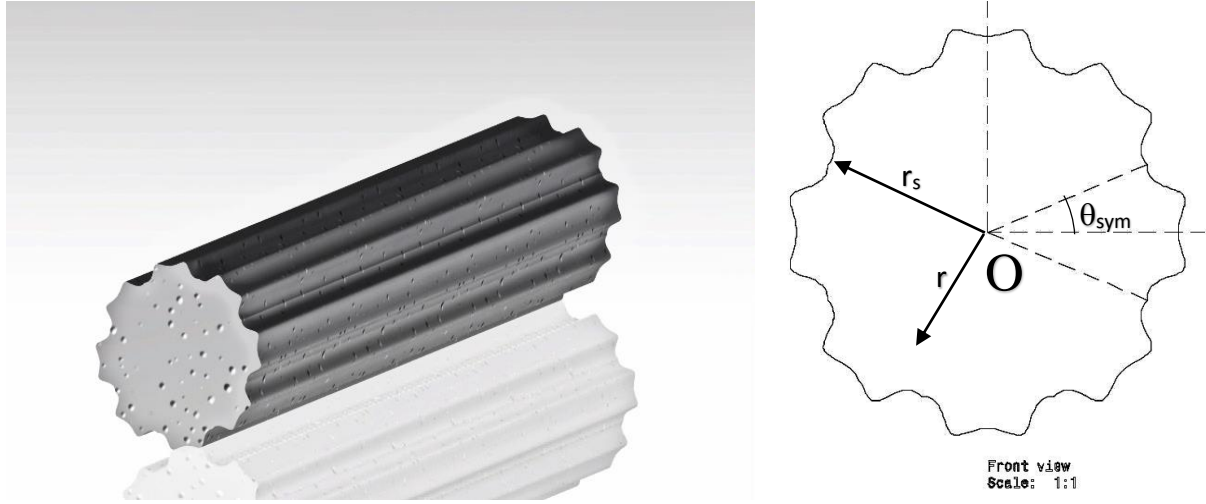


## **Generalization of stress function equations:**

**Case I:** Solid bar with arbitrary outer periphery.

Polar coordinate system  $(r, \theta, z)$  is selected for formulating the problem mathematically. The formulation presented is generalized and it can be applied for to any cross-section provided, they have at least one plane of symmetry in a direction perpendicular to that of the cross-section of the bar. This status is achieved when the angle included in the cross-sectional segment is a factor of  $2\pi$  and the factor is an integer  $N$ . The shape of the bar within this symmetric segment can be completely arbitrary, as is shown in Fig. 1.

Consider this cross section shown in Fig. 1 below:



*Figure 1 A bar with arbitrary cross section*

The torsion problems in prismatic bars are developed in terms of the stress function, which fulfills the Poisson's equation

$$\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r} \frac{\partial \Psi}{\partial r} + \frac{\partial^2 \Psi}{\partial \theta^2} = -2\mu\omega \quad (1)$$

where  $\Psi$  is the stress function, shear modulus of rod material is given by  $\mu$  and angle of twist of the rod per unit length is given by  $\omega$ . The location of bar periphery with respect to the point O is described by  $r_s$  which is a function of  $\theta$  where  $\theta$  varies from 0 to  $\theta_{\text{sym}}$ .

Now we introduce non dimensional variables which are of the form:

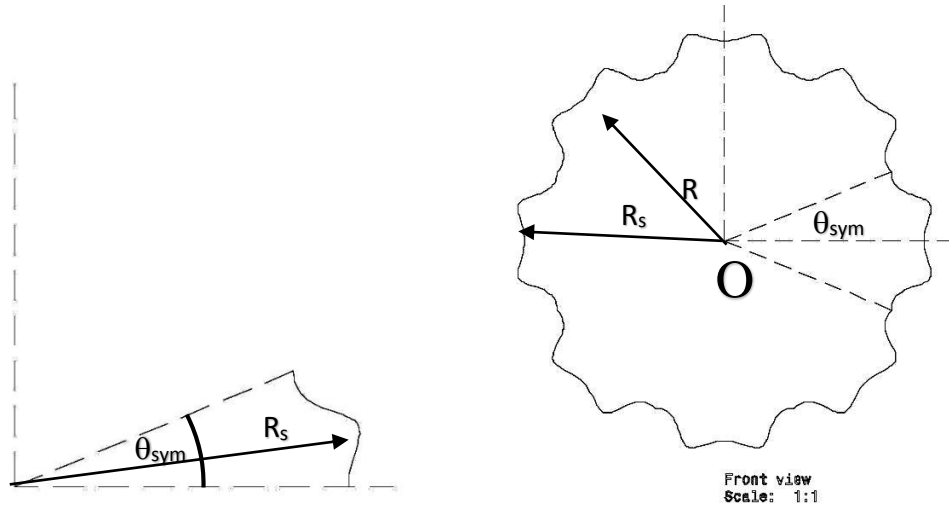
$$R = \frac{r}{\ell}, R_s = \frac{r_s}{\ell}, \psi = \frac{\Psi}{2\mu\omega\ell^2} \quad (2)$$

where ‘ $\ell$ ’ is any length scale that is required so as to non-dimensionalize. Length scales to be considered based on the geometry i.e. varying from one geometry to another geometry; this has to be fixed beforehand by inspection. Now the governing equation has the form:

$$\frac{\partial^2 \psi}{\partial R^2} + \frac{1}{R} \frac{\partial \psi}{\partial R} + \frac{\partial^2 \psi}{\partial \theta^2} = -1 \quad (3)$$

The general solution for this governing equation is of the form:

$$\begin{aligned} \psi(R, \theta) = & -\frac{R^2}{4} + B \ln R + C\theta + D\theta \ln R \\ & + \sum_{k=1}^{\infty} (A_k R^{\lambda_k} + B_k R^{-\lambda_k}) \cos(\lambda_k \theta) \\ & + \sum_{k=1}^{\infty} (C_k R^{\lambda_k} + D_k R^{-\lambda_k}) \sin(\lambda_k \theta) \end{aligned} \quad (4)$$



*Figure 2 Symmetric segment of the arbitrary cross section.*

The  $\theta_{sym}$  Shown in the Fig. 2 is the angle of symmetry and a symmetric segment has also been shown. The Fig. 2 shows a bar of arbitrary cross section and the figure alongside show one of its

symmetric angular segment ranging from  $\theta = 0$  to  $\theta = \theta_{\text{sym}}$ . The boundary conditions in this case are:

$$\frac{\partial \psi}{\partial \theta} = 0 \quad \text{at } \theta = 0 \quad (5)$$

$$\frac{\partial \psi}{\partial \theta} = 0 \quad \text{at } \theta = \theta_{\text{sym}} = \pi/N \quad (6)$$

$$\psi = 0 \quad \text{at } R = R_s \quad (7)$$

The solution of the above equation when subjected to boundary conditions as in Eq. (5) and Eq. (6), we get an equation similar to:

$$\psi(R, \theta) = -\frac{R^2}{4} + \sum_{k=1}^{\infty} \left( Y_k R^{N(k-1)} \cos[N(k-1)\theta] \right) \quad (8)$$

where  $Y_k$  are the unknown coefficients.

Now, this  $Y_k$  can be found by using the boundary condition as in Eq. (7).

At  $R = R_s$ , Eq. (8) becomes:

$$\frac{R_s^2}{4} = \sum_{k=1}^{\infty} \left( Y_k R_s^{N(k-1)} \cos[N(k-1)\theta] \right) \quad (9)$$

In this equation, the unknowns are  $Y_k$ , where  $k$  varies from 0 to  $\infty$ . Nonetheless, the Eq. (9) cannot be assessed certainly because of uniqueness in geometry of the outer surface which has any arbitrary cross-section ( $r_s$  is a function of  $\theta$ ). In order to solve this problem, the BCM is applied at a finite number of collocation points which are equidistant to each other and are present on the arbitrary boundary. This arbitrarily shaped surface is defined uniquely by  $r_s(\theta)$  within the limits  $\theta = 0$  and  $\theta = \pi/N$ . Now, the infinite series which is represented in Eq. (9) is cut short to the first  $M$  number of terms and Eq. (9) is satisfied at the collocation points considered on the non-circular boundary. Hence we get  $M$  number of unknowns ( $Y_k$ ,  $k = 1$  to  $M$ ) and an equivalent number of linear equations. So we may write Eq. (9) as a set of linear equations, the unknown constants being represented by  $Y_k$ , given by,

$$\sum_{k=1}^M (G_{jk} Y_k) = F_j \quad j=1,2,3 \dots M \quad (10)$$

where,

$$G_{jk} = R_s^{N(k-1)} \cos[N(k-1)\theta] \quad (11)$$

$$F_j = \frac{R_s^2}{4} \quad (12)$$

Now M values of  $\theta$  have to be chosen and the selection of the values which again depends on the problem. Angles were taken such that the points are equidistant from each other. The system of linear equations mentioned above, which is shown in Eq. (10), may be evaluated to determine the values of the constants that are unknown i.e.  $Y_k$ . Hence the complete stress function profile is obtained and it is given by Eq. (8). After applying the BCM, we find out the values of coefficients  $Y_1, Y_2, \dots, Y_M$  and hence we get the formula for the stress function  $\psi$  explicitly, i.e.

$$\psi(R, \theta) = -\frac{R^2}{4} + \sum_{k=1}^M \left( Y_k R^{N(k-1)} \cos[N(k-1)\theta] \right) \quad (13)$$

In order to arrive at a reasonably accurate solution, the number of points that has to be taken depends on the complicity of the arbitrary shape. Now, in order to find non-dimensional stiffness of rods, we have to evaluate twisting moment in rods. Then the stress functions are integrated analytically and a formula for the same is also found analytically for all the cases. Hence, torsion in prismatic rods is given by:

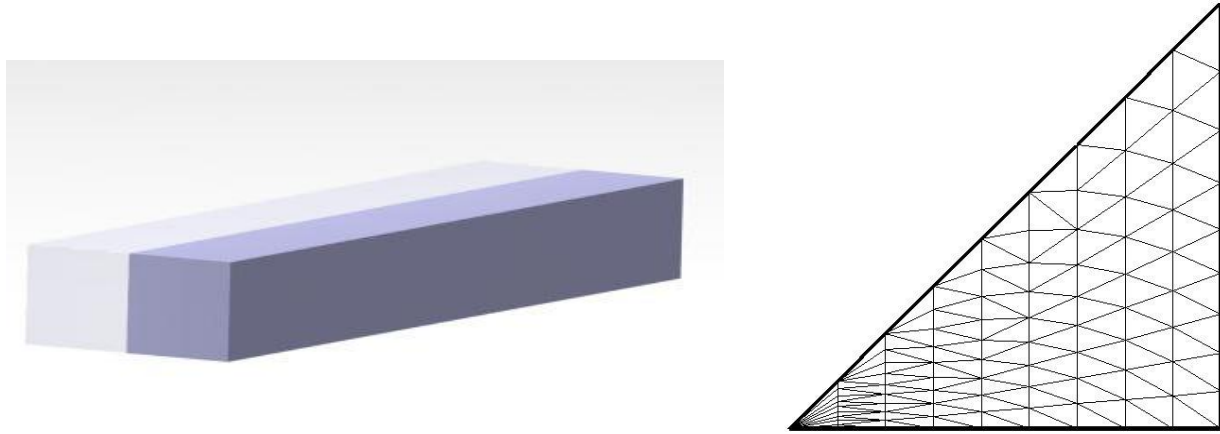
$$M = 2 \int_A \psi dA \quad (14)$$

where A is area of cross-section of the rod. The non-dimensional torsional stiffness of the rods have the form:

$$D = \frac{M}{\mu \omega a^4} \quad (15)$$

For the case 1, i.e. a solid arbitrary cross section, two such examples have been considered.

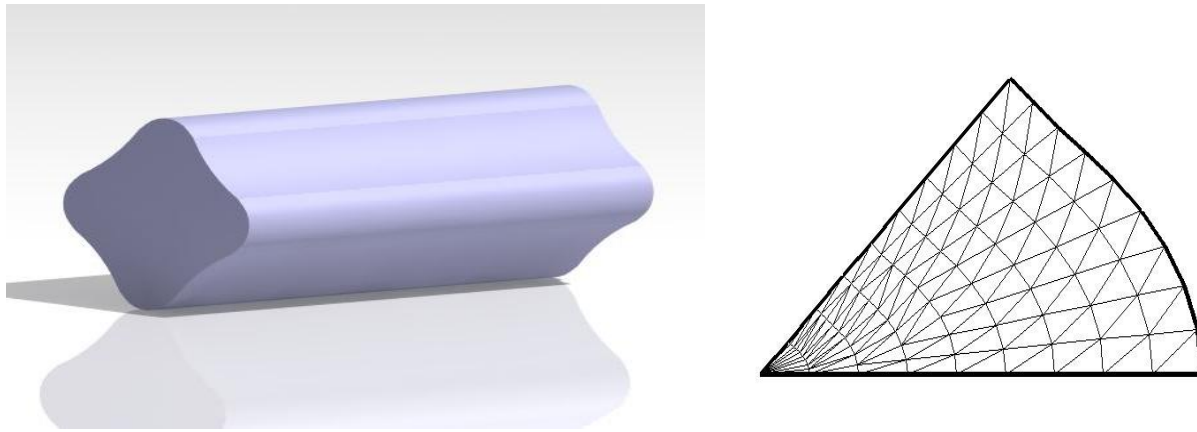
**Example 1:** Cross-section of the bar is a square. The bar of square cross section is shown below:



*Figure 3 Bar of square cross section and its symmetric segment.*

Now the equation of outer periphery in its angular symmetric segment ranging from  $\theta = 0$  to  $\theta = \theta_{\text{sym}}$  is given by:  $R_s = 1/\cos(\theta)$ . No of symmetric segments  $N = 4$ .

**Example 2:** Cross-section of the bar whose outer periphery is defined by the equation  $0.9 + 0.1\cos(4\theta)$  (40). A bar of similar cross section has been shown below:

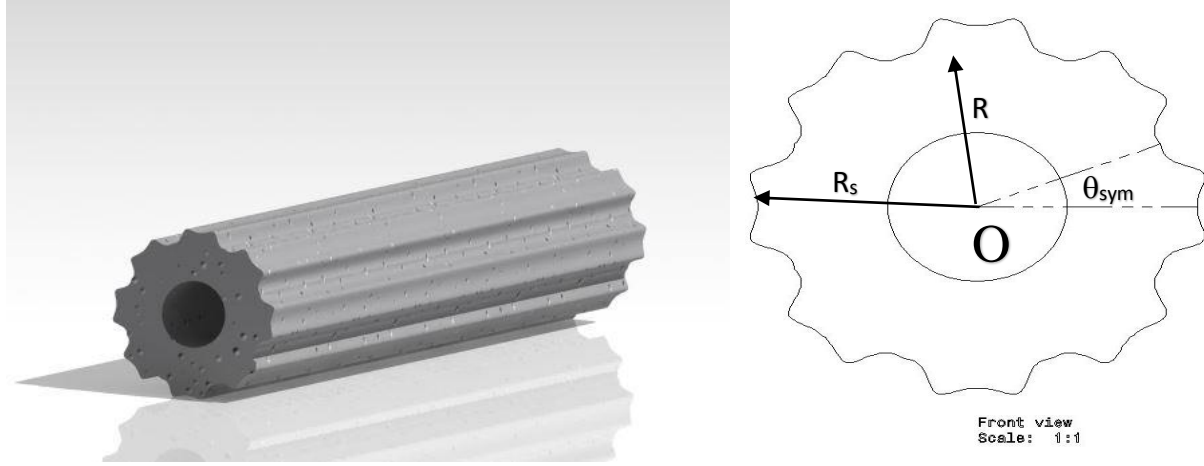


*Figure 4 Bar of arbitrary cross-section and its symmetric segment*

$R_s = 0.9 + 0.1\cos(4\theta)$ . No of symmetric segments  $N = 4$ .

The variation of stress function throughout the cross section was found out by incorporating the method explained earlier in a MATLAB program (refer Appendix A). The coordinates were obtained and the corresponding value of the stress function was obtained. A contour plot was made and plot has been shown later.

**Case 2:** Bars with arbitrary outer periphery and a circular centered holes.

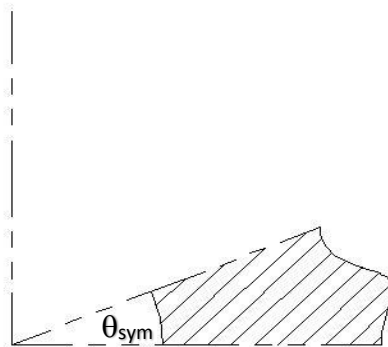


*Figure 5 Bar of arbitrary outer periphery with cylindrical hole and its front view*

A bar of arbitrary cross-sectional shape having a circular centered hole is shown in Fig. 5 along with the front view of the bar.

The non-dimensional variables required for the geometry (taking  $\ell = r_i$ ) are as given as:

$$R = r/r_i \ ; \quad R_s = r_s / r_i \quad (16)$$



*Figure 6 Symmetric segment of the bar*

such that for the current geometry,  $1 \leq R \leq R_s$ . A symmetric segment of this kind of cross section has been shown in the Fig. 6.

The boundary conditions in a non-dimensional form are as given by:

$$\frac{\partial \psi}{\partial \theta} = 0 \quad \text{at } \theta = 0 \quad (17)$$

$$\frac{\partial \psi}{\partial \theta} = 0 \quad \text{at } \theta = \theta_{\text{sym}} = \pi/N \quad (18)$$

$$\psi = 0 \quad \text{at } R = R_s \quad (19)$$

$$\psi = \psi_0 \quad \text{at } r = r_i \text{ or } R = 1 \quad (20)$$

where, the value of  $\psi_0$  is chosen such that it satisfies the following condition:

$$\int_0^{2\pi} \frac{\partial \psi}{\partial \theta} \bigg|_{R=1} d\theta = -\pi \quad (21)$$

If we consider the same approach as in case 1, we get the stress function as:

$$\psi(R, \theta) = -\frac{(R^2 - 1)}{4} + Y_1 + \sum_{k=2}^{\infty} \left( Y_k \left( R^{N(k-1)} - \frac{1}{R^{N(k-1)}} \right) \cos[N(k-1)\theta] \right) \quad (22)$$

where the unknown coefficients are  $Y_k$ .

Now, this  $Y_k$  can be found by using the boundary condition as in Eq. (19).

At  $R = R_s$ , Eq. (22) becomes:

$$\frac{(R_s^2 - 1)}{4} = Y_1 + \sum_{k=2}^{\infty} \left( \left( Y_k \left( R_s^{N(k-1)} - \frac{1}{R_s^{N(k-1)}} \right) \right) \cos[N(k-1)\theta] \right) \quad (23)$$

Now  $M$  values of  $\theta$  have to be chosen and the selection of the values which again depends on the problem. Angles were taken such that the points are equidistant from each other. After obtaining the values of  $\theta$ , and dropping the values for  $k > M$  the infinite series to first  $M$  terms, we get a set of linear equations which helps in determining the values of  $Y_k$ ,

$$\sum_{k=1}^M (G_{jk} Y_k) = F_j \quad j=1,2,3 \dots M \quad (24)$$

where,

$$G_{j1} = 1, \quad (25)$$

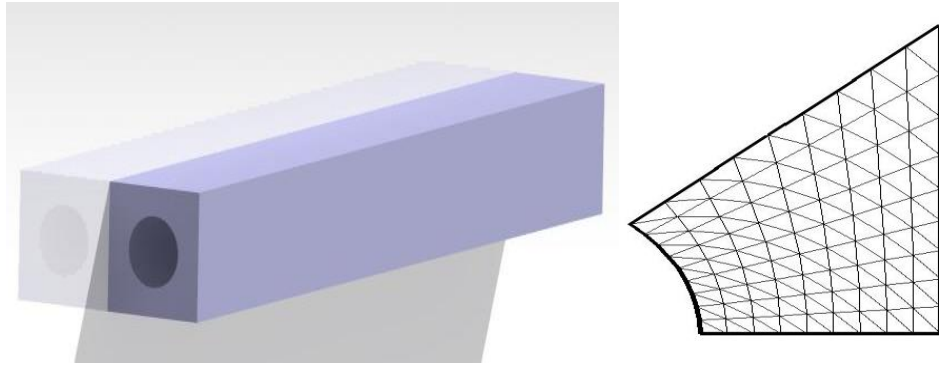
$$G_{jk} = \left( R_s^{N(k-1)} - \frac{1}{R_s^{N(k-1)}} \right) \cos[N(k-1)\theta] \quad (26)$$

$$F_j = \frac{R_s^2 - 1}{4} \quad (27)$$

Putting the values of  $Y_k$  in Eq. (22), we find the value of stress function throughout the cross section.

For the case 2, i.e. bars with arbitrary outer periphery and a circular centered holes, two such examples have been taken.

**Example 3:** Outer periphery of the bar is a square with a circular centered hole. The bar of square cross section is shown below:



*Figure 7 Bar of square cross section with circular hole and its symmetric segment*

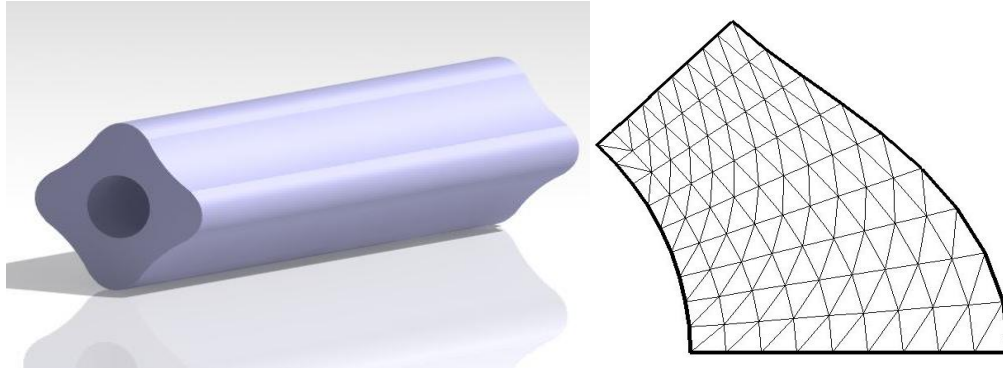
Now the equation of outer periphery in its angular symmetric segment ranging from  $\theta = 0$  to  $\theta = \theta_{\text{sym}}$  is given by

$$R_s = 2/\cos(\theta).$$

No of symmetric segments  $N = 4$ .



**Example 4:** Cross-section of the bar whose outer periphery is defined by the equation  $0.9 + 0.1\cos(4\theta)$ . A bar of similar cross section has been shown below:



*Figure 8 Bar with arbitrary cross section with a cylindrical hole and its symmetric segment*

$$R_s = 0.9 + 0.1\cos(4\theta).$$

No of symmetric segments  $N = 4$ .

The variation of stress function throughout the cross section was found out by incorporating the method explained earlier in a MATLAB program (refer Appendix B). The coordinates were obtained and the corresponding value of the stress function was obtained a contour plot was made and plot had been shown later.

**Case 3:** Cylindrical bars with arbitrary holes.

A bar of circular cross-section which has any arbitrarily shaped hole is shown in Fig. 9 below along with the front view of the bar.



*Figure 9 Cylindrical bar with arbitrary centered hole and its front view*

The non-dimensional variables required for the above geometry (taking  $\ell = r_i$ ) are as follows:

$$R = r / r_i; R_s = r_s / r_i \quad (28)$$

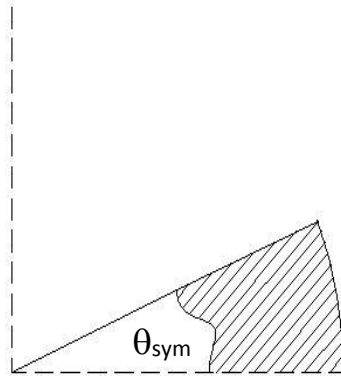


Figure 10 Symmetric segment of the bar

such that for the current geometry,  $1 \leq R \leq R_s$ . A symmetric segment of this kind of cross section has been shown in the Fig. 10

The boundary conditions in a non-dimensional form are given by:

$$\frac{\partial \psi}{\partial \theta} = 0 \quad \text{at } \theta = 0 \quad (29)$$

$$\frac{\partial \psi}{\partial \theta} = 0 \quad \text{at } \theta = \theta_{sym} = \pi/N \quad (30)$$

$$\psi = 0 \quad \text{at } R = R_s \quad (31)$$

$$\psi = \psi_0 \quad \text{at } r = r_i \text{ or } R = 1 \quad (32)$$

where, the value of  $\psi_0$  is chosen such that it satisfies the following condition:

$$\int_0^{2\pi} \frac{\partial \psi}{\partial \theta} \bigg|_{R=R_s} d\theta = -\pi \quad (33)$$

If we consider the same approach as in case 1, we get the stress function as:

$$\psi(R, \theta) = -\frac{(R^2 - 1)}{4} + Y_1 + \sum_{k=2}^{\infty} \left( Y_k \left( R^{N(k-1)} - \frac{1}{R^{N(k-1)}} \right) \cos[N(k-1)\theta] \right) \quad (34)$$

where  $Y_k$  are the unknown coefficients.

Now, this  $Y_k$  can be found by using the boundary condition (31).

At  $R = R_s$ , Eq. (34) becomes:

$$\frac{(R^2 - 1)}{4} = Y_1 + \sum_{k=2}^{\infty} \left( Y_k \left( R_s^{N(k-1)} - \frac{1}{R_s^{N(k-1)}} \right) \cos[N(k-1)\theta] \right) \quad (35)$$

Now  $M$  values of  $\theta$  have to be chosen and the selection of the values which again depends on the problem. Angles were taken such that the points are equidistant from each other. After obtaining the values of  $\theta$ , and dropping the values for  $k > M$ , we make a finite series having  $M$  terms, and we get a set of linear equations which helps in determining the values of  $Y_k$ ,

$$\sum_{k=1}^M (G_{jk} Y_k) = F_j \quad j = 1, 2, 3 \dots M \quad (36)$$

where,

$$G_{j1} = 1, \quad (37)$$

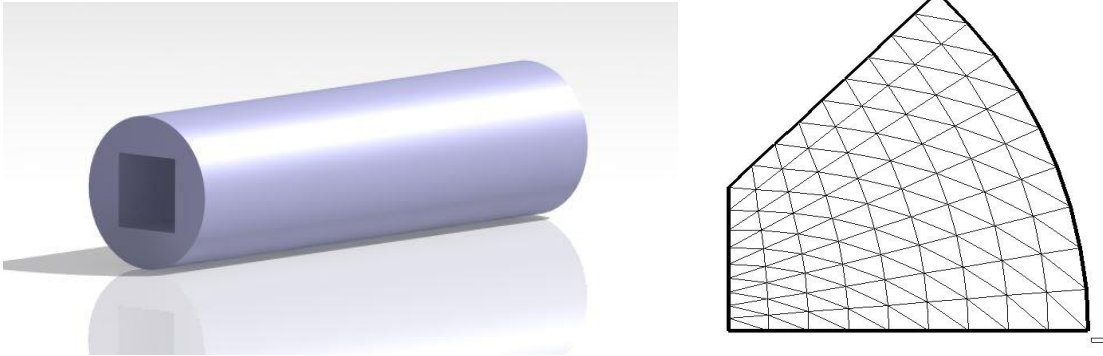
$$G_{jk} = \left( R_s^{N(k-1)} - \frac{1}{R_s^{N(k-1)}} \right) \cos[N(k-1)\theta] \quad (38)$$

$$F_j = \frac{R_s^2 - 1}{4} \quad (39)$$

Putting the values of  $Y_k$  in Eq. (34), we find the value of stress function throughout the cross section.

For the case 3, cylindrical bars with arbitrary holes, two such examples have been considered.

**Example 5:** Outer periphery of the bar is circular with a square centered hole. The bar of this kind of cross section is shown below:



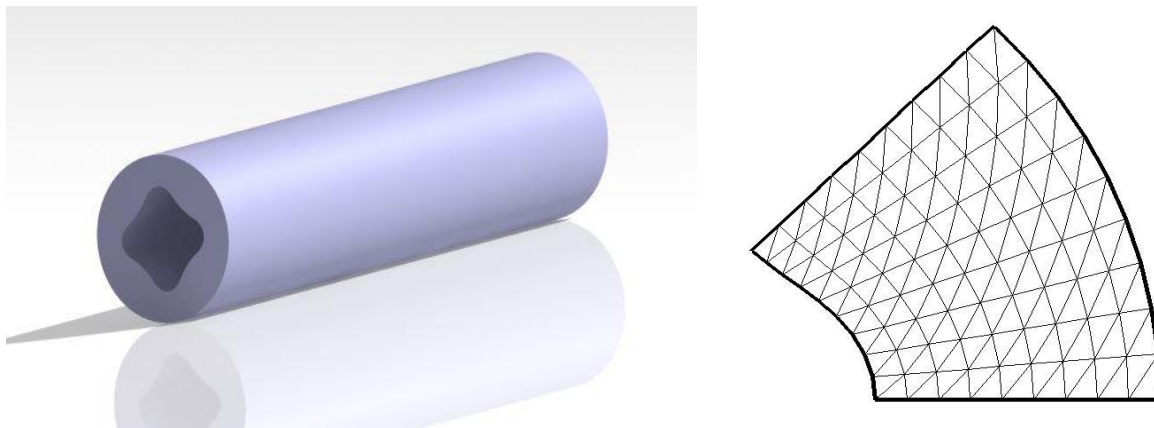
*Figure 11 Bar of square cross section with circular hole and its symmetric segment*

Now the equation of outer periphery in its angular symmetric segment ranging from  $\theta = 0$  to  $\theta = \theta_{\text{sym}}$  is given by

$$R_s = 1/\cos(\theta).$$

No of symmetric segments  $N = 4$ .

**Example 6:** Cross-section of a cylindrical bar whose hole is defined by the equation  $(0.9/2) + (0.1/2)\cos(4\theta)$ . A bar of similar cross section has been shown below:



*Figure 12 Bar of arbitrary cross section with circular hole and its symmetric segment*

$$R_s = 0.9 + 0.1\cos(4\theta).$$

No of symmetric segments  $N = 4$ .

The variation of stress function throughout the cross section was found out by incorporating the method explained earlier in a MATLAB program (refer Appendix C). The coordinates were obtained and the corresponding value of the stress function was obtained and a contour plot was made and plot had been shown later.

## **Results and discussion:**

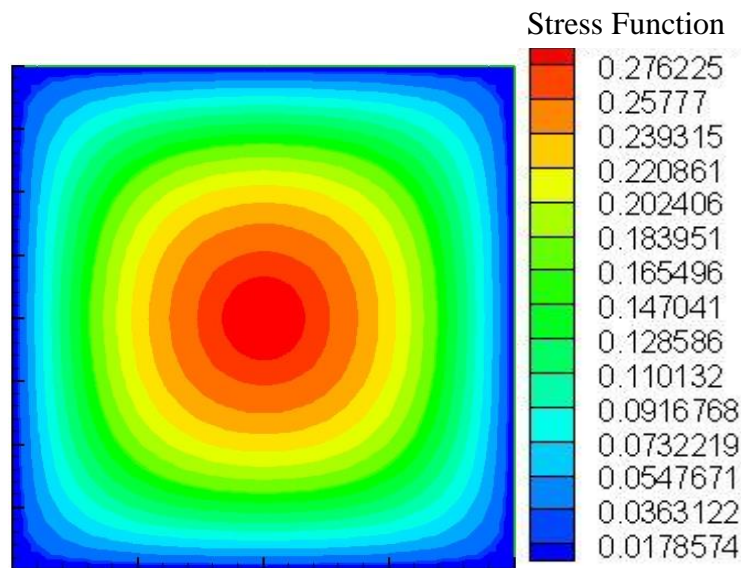
In the first case where we had taken a solid cross section, two examples were taken to illustrate the working of the MATLAB code. Using MATLAB, (x,y) coordinates were obtained for the profile and stress function values of various points were calculated and a contour plot was made, and it was observed that at the periphery, because of absence of any force, stress function value is minimum (theoretically zero) which is shown by the smaller values in stress function. As we approach towards the center, that value increases. The value is maximum at the center.

### **Example 1:**

$$R = 1/\cos(\theta).$$

$$\theta_{\text{sym}} = 45^\circ.$$

No of symmetric segments = 4



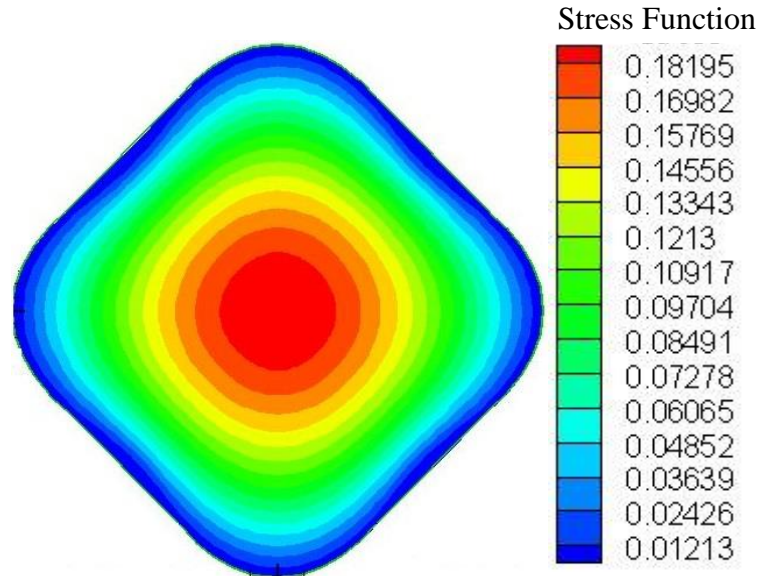
*Figure 13 Stress function for bar of square cross-section*

### **Example 2:**

$$R_s = 0.9 + 0.1\cos(4\theta).$$

$$\theta_{\text{sym}} = 45^\circ.$$

No of symmetric segments=4



*Figure 14 Stress function for a bar of arbitrary cross section*

In the second case where we had taken an arbitrary cross section with a circular hole, two examples were taken to illustrate the working of the MATLAB code. Using MATLAB, (x,y) coordinates were obtained for the profile and stress function values at various points were calculated and a contour plot was made, and it was observed that at the periphery, because of absence of any force, stress function value is minimum (theoretically zero) which is shown by smaller values in stress function. As we approach towards the periphery of the hole, the stress function value increases. The value is maximum at the inner periphery i.e. periphery of the hole.

### **Example 3:**

$$R_s = 1/\cos(\theta).$$

$$R_i = 1$$

$$\theta_{\text{sym}} = 45^\circ.$$

No of symmetric segments = 4

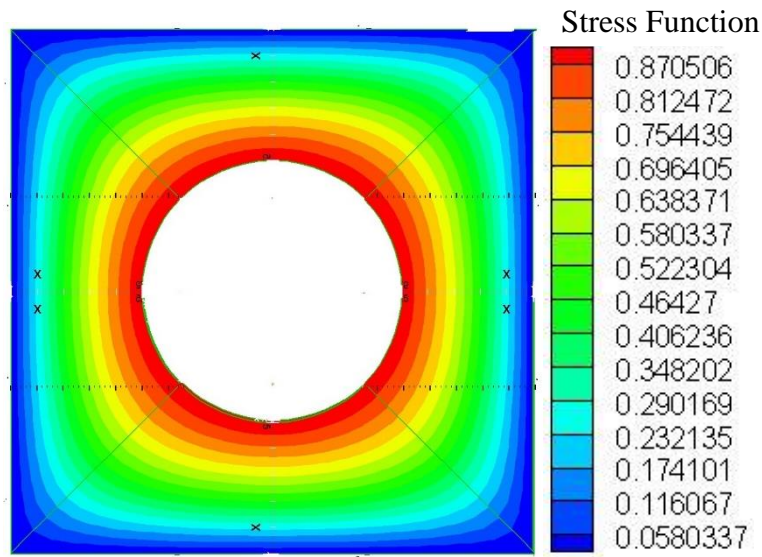


Figure 15 Stress function for square bar with cylindrical hole

**Example 4:**

$$R_s = 0.9 + 0.1\cos(4\theta).$$

$$R_i = 1.$$

$$\theta_{\text{sym}} = 45^\circ.$$

No of symmetric segments = 4

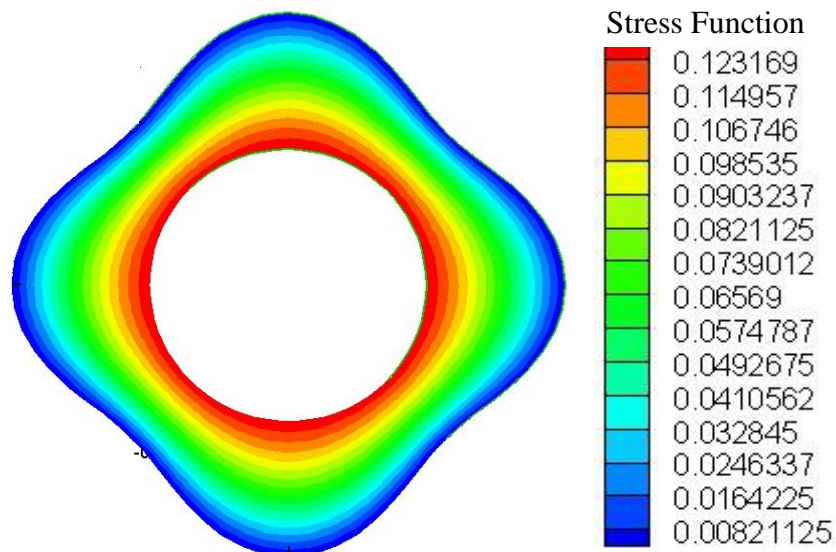


Figure 16 Stress function for arbitrary bar with cylindrical hole



In the third case where we had taken a circular cross section with an arbitrary hole, two examples were taken to illustrate the working of the MATLAB code. Using MATLAB, (x,y) coordinates were obtained for the profile and stress function values at various points were calculated and a contour plot was made, and it was observed that at the periphery, because of absence of any force, stress function value is minimum which is shown by negligible values of stress function. As we approach towards the periphery of the hole, the stress function value increases. The value is maximum at the inner periphery i.e. periphery of the hole.

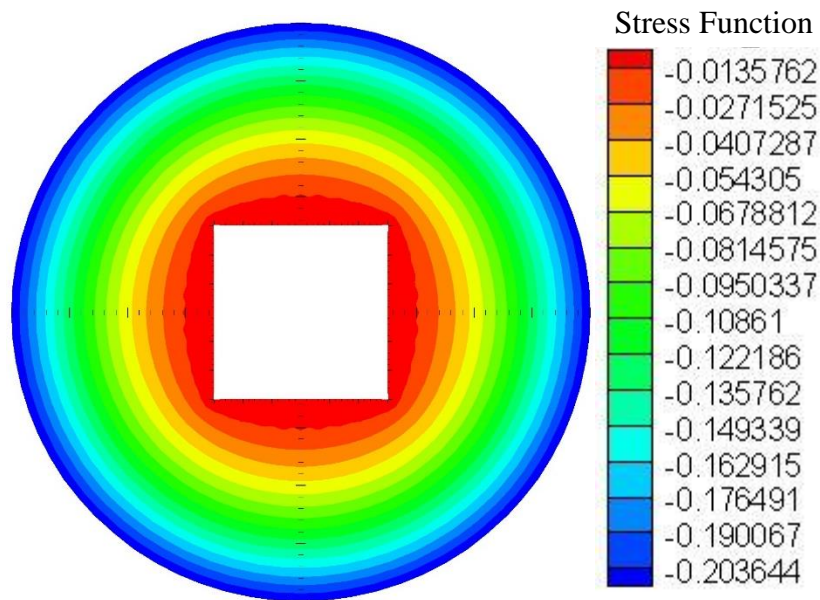
**Example 5:**

$$R_s = 1/\cos(\theta).$$

$$R_o = 1$$

$$\theta_{\text{sym}} = 45^\circ.$$

No of symmetric segments = 4



*Figure 17 Stress function in cylindrical bar with a square hole*



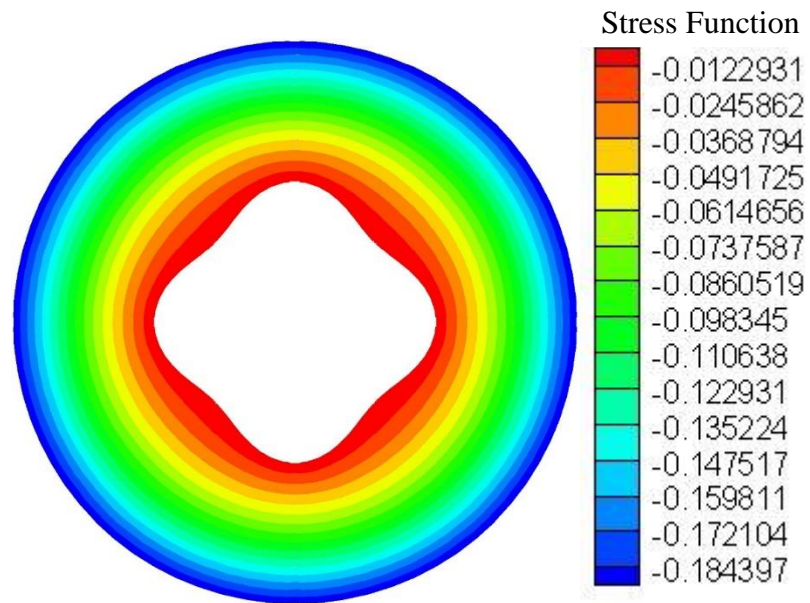
**Example 6:**

$$R_s = (0.9/2) + (0.1/2)\cos(4\theta)$$

$$R_o = 1$$

$$\theta_{\text{sym}} = 45^\circ.$$

No of symmetric segments = 4



*Figure 18 Stress function in cylindrical bar with arbitrary hole*

## **Conclusion:**

This work involves torsion in rods whose cross-section is arbitrary in nature. The objective of this work was to generalize a solution that can be applied for bars having any kind of cross-section, with or without holes of any arbitrary shape, provided that, it has at least one line of symmetry. The solution was to be incorporated in a MATLAB code, which would take basic inputs like equations defining cross-sectional shape, and the results were to be plotted.

In this paper, the governing differential equations were solved in MATLAB after simplification by using boundary conditions. Hence a code has been written that is generalized for any arbitrary shape. All that needs to be done is to edit the code only where the boundary defining equation is written, rest of the code remains the same. Using the code, we find out the cartesian coordinates of the cross section as well as the corresponding values of the stress function, those values can be represented in a contour plot to visualize the way the stress function varies across the cross section.

## **References:**

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2. Kolodziej, J. A., and Fraska, A., 2005, “Elastic torsion of bars possessing regular polygon in cross-section using BCM”, *Computers and Structures*, 84(1-2), 78–91.
3. Moharana, M. K., and Das, P. K, 2008 “Heat conduction through heat exchanger tubes of noncircular cross section,” *J. Heat Transfer* 130(1), pp. 1-6.
4. Moharana, M. K., and Das, P. K, 2012, “Heat conduction through eccentric annuli: an appraisal of analytical, semi-analytical, and approximate techniques,” *J. Heat Transfer* 134(9), pp. 1-6.
5. Moharana, M. K., and Khandekar, S, 2013, “Generalized formulation for estimating pressure drop in fully-developed laminar flow in singly and doubly connected channels of non-circular cross-sections,” *Computer Methods in Applied Mechanics and Engineering*, 259(1), pp. 64-76.

## Appendix A

***MATLAB code for calculation of stress function of the bar with square cross section:***

```

prompt='enter no of points';
M=input(prompt); %no of collocation points
L=4;
N=M; %no of linear divisions
Th=0;
for j=1:M

Rs=1/cos(Th); % boundary coordinates that depends on
arbitrariness of function

for k=1:M
    X(j,k)=(Rs^(L*(k-1)))* cos(L*Th*(k-1));
    Z(j,1)=(Rs^2)/4;
end
Th=Th+(pi/L)/(M-1);

end

Y=X\Z;
Th=0;

for i=1:M

    r=0;
    for j=1:N
        Rs=1/cos(Th);
        O=-(r^2)/4;
        for k=1:M
            O=O+(Y(k)*(r^(L*(k-1)))* cos(L*Th*(k-1)));
        end
        a=r*cos(Th);
        b=r*sin(Th);
        X=[num2str(a), ' ', num2str(b), ' ', num2str(O)];
        disp(X);
        r=r+Rs/(N-1);

    end;
    Th=Th+(pi/L)/(M-1);
end

```

## Appendix B

### ***MATLAB code for bar with square cross section with cylindrical hole:***

```

prompt='enter no of points';
M=input(prompt);
L=4; % number of symmetry elements
N=M; %no of linear divisions
Th=0;
for j=1:M

Rs=2/cos(Th);    %enter Boundary equation as a function of Theta
X(j,1)=1;
    for k=2:M
        X(j,k)=(((Rs^(L*(k-1))))-1/((Rs^(L*(k-1)))))*cos(L*Th*(k-1));
        Z(j,1)=(Rs^2-1)/4;
    end
Th=Th+(pi/L)/(M-1);
end
Y=X\Z;
Th=0;
    for i=1:M

r=1;
        for j=1:N
            Rs=2/cos(Th);    %enter Boundary equation as a function of
Theta

            O=- (r^2-1)/4+Y(1);
            for k=2:M
                O=O+(Y(k)*(((r^(L*(k-1))))-1/((r^(L*(k-1)))))*cos(L*Th*(k-1)));
            end

            a=r*cos(Th);
            b=r*sin(Th);

            X=[num2str(a), ' ', num2str(b), ' ', num2str(O)];
            disp(X);

            r=r+(Rs-1)/(N-1);
        end
Th=Th+(pi/L)/(M-1);

end

```

## Appendix C

***MATLAB code for cylindrical bar with arbitrary shaped hole.***

```

prompt='enter no of points';
M=input(prompt);
L=4; % number of symmetry elements
N=M; %no of linear divisions
Th=0;
    for j=1:M

        Rs=(0.9/2)+(0.1/2)*cos(4*Th);%enter Boundary equation
as a function of Theta
        X(j,1)=1;
            for k=2:M
                X(j,k)=(((Rs^(L*(k-1))))-1/((Rs^(L*(k-
1)))))*cos(L*Th*(k-1));
                Z(j,1)=(Rs^2-1)/4;
                    end
                Th=Th+(pi/L)/(M-1);
            end
        Y=X\Z;
        Th=0;
        for i=1:M

            r=(0.9/2)+(0.1/2)*cos(4*Th);
            for j=1:N
                Rs=(0.9/2)+(0.1/2)*cos(4*Th);
                O=-(r^2-1)/4+Y(1);
                for k=2:M
                    O=O+(Y(k)*(((r^(L*(k-1))))-1/((r^(L*(k-
1)))))*cos(L*Th*(k-1)));
                end

                a=r*cos(Th);
                b=r*sin(Th);

                X=[num2str(a),' ',num2str(b),'
',num2str(O)];
                disp(X);

                r=r+(1-Rs)/(N-1);
            end
            Th=Th+(pi/L)/(M-1);
        end
    end
end

```

## Appendix D

### **Variable declaration for MATLAB codes:**

Variable Name	Significance
M	Number of collocation points M
L	Number of symmetric elements
N	Number of linear divisions
Th	Angle $\theta$
r	Radius r
i	Loop variable
j	Loop variable
k	Loop variable
Rs	Represents the arbitrary periphery $R_s$
X( )	An array that stores the coefficients of $Y_k$
Y( )	An array that stores the values of $Y_k$
Z( )	An array that stores the constant in a set of linear equations
O	Stores the value of stress function
a	Stores x coordinate of any point (r, $\theta$ )
b	Stores y coordinate of any point (r, $\theta$ )
X	Stores values for proper display